AN OPTIMUM STIFFENER ARRANGEMENT FOR LONGITUDINALLY STIFFENED PLATES SUBJECTED TO LINEARLY VARYING AXIAL AND LATERAL LOADS

Brian J. Rabe

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by

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Abstract

Title: An Optimum Stiffener Arrangement for Longitudinally Stiffened Plates Subjected to Linearly Varying Axial and Lateral Loads.

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Designers are constantly faced with the problem of designing a product to meet some minimum standard. In the area of ship and aircraft design it is also imperative that a product is not over-designed. An optimum design will just meet all structural requirements with no added material or weight. This thesis addresses the problem of an optimum or least weight plate-stiffener arrangement for a typical panel of side shell plating for a ship. This type of panel is subjected to a linearly varying lateral load due to a head of seawater or cargo. With the use of a common interaction formula to relate the axial and lateral loads a set of design curves is developed which aid in determining an optimum plate-stiffener arrangement. The optimum design will have varying stiffeners and spacings as dictated by the varying loads. As varying stiffeners and spacings may not be practical from a fabrication standpoint, methods for using the curves employing one stiffener and one stiffener spacing are also described. Finally included is a description of a computer program which was written to solve a form of the interaction formula for the various variables.

Thesis Supervisor: J.H. Evans
Title: Professor of Naval Architecture

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I. Introduction

The problem of designing ship and aircraft structures for minimum weight has been one of considerable interest to designers for several years. The idea is very straightforward; by placing certain structural elements in proper configurations an optimum or minimum structural weight design may be attained. The minimum structural weight solution is desireable as it allows the ship or aircraft to carry more payload weight for a given volume, and it saves money by reducing the amount of structural material.

In the area of shell plating for ship structures, two theses done at M.I.T. have treated optimum longitudinal (stiffened parallel to applied axial load) stiffener arrangements. The first of these, by Harlander (1), deals with a plate subjected to two separate loading conditions, a uniform axial load and a uniform lateral load. The second thesis, by Lyons and Webb (2), treats the stiffener arrangement for a plate subjected to combined uniform axial and lateral loads. This paper tackles the combined loading problem of least weight stiffener arrangement with one added dimension. Instead of the plate being subjected to combined uniform loads the plate under investigation here has combined linearly varying axial and lateral loads. The side shell plating of a ship experiences this type of loading. The linearly varying axial load results from the ship bending as a beam, and the linearly varying lateral load may result from a head of seawater and/or from whatever cargo the ship may be carrying. The studies

previously mentioned, dealing with uniform loads, have specified uniform stiffeners and stiffener spacings for optimum stiffener arrangements. This type of arrangement would not represent a minimum weight solution for a plate with linearly varying loads. This paper describes the development and use of a set of design curves that will yield an optimum stiffener arrangement using various stiffeners and spacings. This type of arrangement with various sizes and spacings may not be practical from a fabrication standpoint. The same design curves may be used to select an optimum solution for a given loading using constant stiffener spacings with various stiffeners, and using one stiffener at various spacings. In this study no attempt is made to distinguish which arrangement is optimal from a fabrication standpoint; the optimum stiffener arrangement hypothesized is simply the one of least weight.

The question may arise as to why a plate subjected to this type of loading should not be stiffened with stiffeners running perpendicular to the axial load. Essentially this would shorten the unsupported span length of each panel allowing a larger axial load. However, with this type of arrangement there would be no method to vary the panel strength with the varying load unless someone develops a tapered stiffener.

It should should be noted here that each set of design curves is useful for panels of only one material, one unsupported span length, and one plate thickness. To generate the curves the designer must know the unsupported span length, and to use them he must know the configuration of the load acting on the plate.

II. PROCEDURE

Although the gross panel considered in this study may have any aspect ratio (length/width) it is assumed that the local panel aspect ratio is greater than 2. (See figure 1.) In this analysis the edges of the local panel elements parallel to the stiffeners are considered clamped and the edges that are loaded are considered simply supported. The lateral loading is assumed to act on the side of the plate without stiffeners. This is an example of an external head of seawater acting against the exterior side shell plating of a ship. This external head compresses the fibers of the plate-stiffener combination which lies toward the plate side of the panel. The outermost fibers in the stiffeners would also have to be investigated if there was any internal lateral load on the panel.

Due to the combined axial and lateral loads on this panel there are two main types of instability by which the panel may fail. The first of these is local instability of the plate, neglecting the action of the stiffeners. However, the stiffeners serve to "clamp" the edges of local panels. The second type of failure is instability of the panel involving column action of the stiffeners.

If any one element of the plate-stiffener combination fails then the whole structure is considered to have failed. Therefore, the optimum distrubution of material (between the plate and stiffener) for maximum buckling strength will occur when the two types

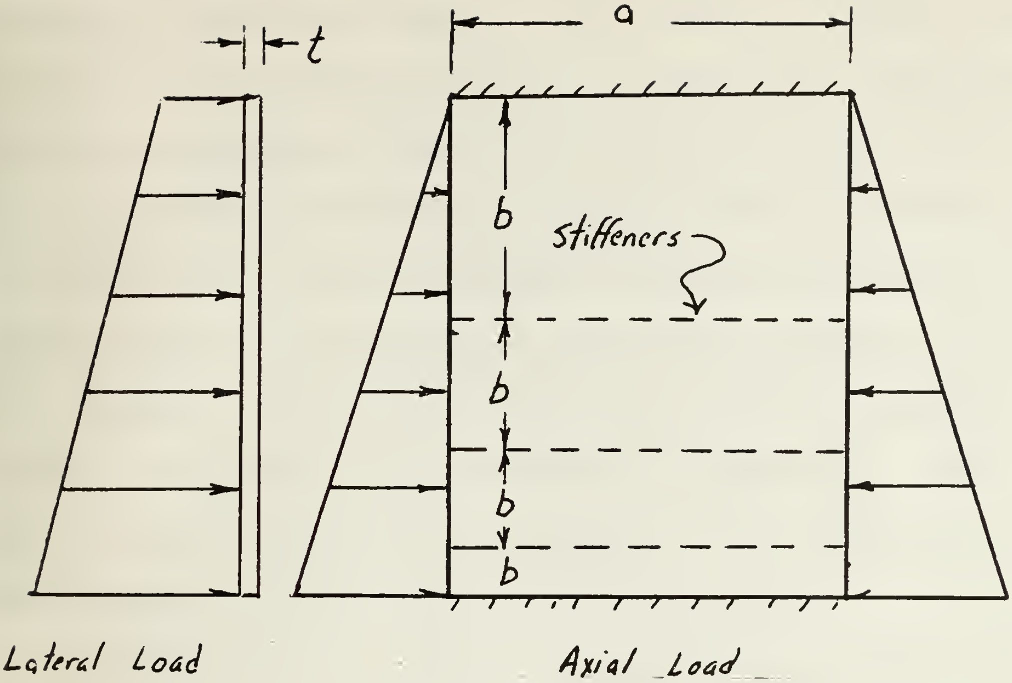
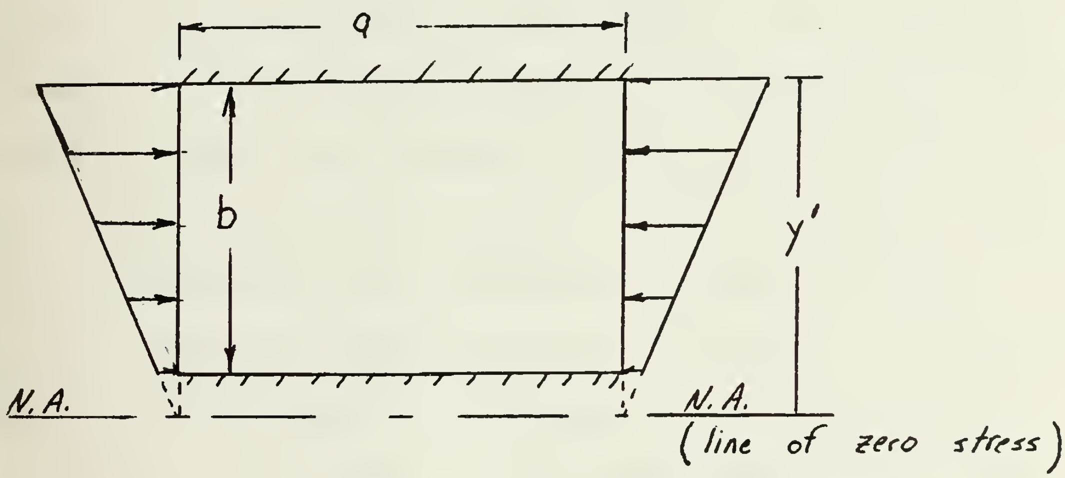


FIG. 1



α = axial load aspect ratio = b/y'

α	0.0	0.2	0.4	0.6	0.8	1.0
K_p	6.97	7.45	8.49	9.67	11.53	13.56

FIG. 2

of instability occur simultaneously. By taking material from a stronger element and adding it to a weaker element the buckling strength of the weaker element is increased, thus increasing the buckling strength of the whole structure. By redistributing the material among the elements until all have the same buckling strength, the buckling strength of the whole structure must be greater than the strength of the element which was originally the weakest. This redistribution of material implies increased buckling strength with no increase in material. This represents the optimum distribution of material (in plate size, stiffener, and spacing) for the structural combination.

Due to the two types of instability by which the panel may fail, a relationship between the two strengths is required. Local instability of the plate is fairly straightforward and will not be discussed here in any detail. The instability of the panel involving column action of the stiffeners requires some investigation and clarification. Several studies have been carried out in this area, the most notable written by Schade (3). These investigations have considered a single stiffener, together with some effective width of plating, acting as a column which fails by bending normal to the plate. Due to the combined lateral and axial loading that the stiffener-plate combination undergoes, the stiffener and its effective width of plating must be treated as a beam-column which, again, fails by bending normal to the plane of the plate. Using this assumption the optimum weight criterion may be accurately stated: "The optimum distri-

bution of material for maximum buckling strength will occur when local buckling of the plate occurs simultaneously with buckling of a stiffener-plate combination treated as a beam-column." (2)

A common interaction formula has been chosen to relate the two types of instability failure.

$$\frac{\sigma_a}{\sigma_{cr\ col}} \times \frac{\sigma_b}{\sigma_{b\ all}} = 1 \qquad \text{or} \qquad \sigma_a = \sigma_{cr\ col}\left[1 - \frac{\sigma_b}{\sigma_{b\ all}}\right] \tag{1}$$

where σ_a = actual applied axial stress (average)

σ_b = actual applied bending stress (maximum)

$\sigma_{cr\ col}$ = limiting column stress, if an axial load alone was being applied

$\sigma_{b\ all}$ = limiting bending stress, if a bending load alone was being applied.

Factors of safety could be incorporated in this formula. Present practice favors the usage of a factor of safety of 1.25 for the axial stress ratio and a factor of safety of about 1.5 for the bending stress ratio. In this study both factors of safety have been set at 1.0 to simplify all calculations.

It is necessary to further define the terms of equation (1). If it is desired that the structure treated as a beam-column fail simultaneously with the local plate panel, then the actual axial load applied must equal the actual plate critical stress. A form of Bryan's equation applies in this case.

$$\sigma_a = \sigma_{cr\ p} = \frac{\pi^2 E\sqrt{\tau_p}}{12(1-\mu^2)} K_p \left(\frac{t}{b}\right)^2 \tag{2}$$

where $\sigma_{cr\ p}$ = plate critical stress

E = modulus of elasticity

τ_p = ratio of tangent modulus (E_t) to Young's modulus (E), $\tau_p = E_t/E$ (τ_p is a function of stress)

μ = Poisson's ratio

t = plate thickness

b = plate breadth (stiffener spacing)

K_p = plate constant, a function of a/b (plate aspect ratio), plate edge conditions, and loading aspect ratio (see figure 2).

If an axial load alone is being applied, the limiting column stress is;

$$\sigma_{cr\ col} = \frac{\pi^2 E\ \tau col}{\left(\frac{K_c a}{r}\right)^2} \tag{3}$$

where τ_{col} is defined as τ_p but will have a different value because stresses are different

Kc = constant determined by the end conditions (=1.0 for simply supported ends)

a = unsupported span length

r = radius of gyration of stiffener-plate combination

The applied bending stress may be written as;

$$\sigma_b = \frac{\gamma H\ b\ a^2}{(K_b Z)144} \tag{4}$$

where γ = specific weight of fluid

H = head of fluid

K_b = constant determined by end conditions (=8.0 for simply supported ends)

Z = section modulus of stiffener-plate combination to extreme plate fiber

Limiting bending stress, $\sigma_{b\ all}$, is equal to yield stress (σ_y) which for mild steel is 33,000 psi.

It has been previously mentioned that the entire amount of plating between stiffeners does not actually act in conjunction with the stiffener when the combination is treated as a column or a beam. The amount of plating acting in conjunction with the stiffener-plate combination is called effective width. Effective width is considered in the radius of gyration (r) of the stiffener plate combination. There are analytical formulas giving the effective width of plating for various conditions, but in this study the effective width used in radius of gyration calculations equals the spacing between stiffeners. It can be seen from figure 3 that this is a conservative assumption because radius of gyration of the stiffener-plate combination actually decreases as the breadth of plate is increased.

The amount of plating between stiffeners that acts effectively with the stiffener in bending is called effective breadth. The term of the interaction formula considering effective breadth is Z, the section modulus of the stiffener-plate combination to the extreme plate fiber. It should be noted that only the section modulus to the extreme plate fiber is of importance in this study, because of the type of lateral loading. The external head of sea-water will tend to place the extreme plate fiber in compression and the extreme stiffener fiber in tension. Since buckling failure occurs as a result of compression, only the section modulus, Z,

FIG. 3

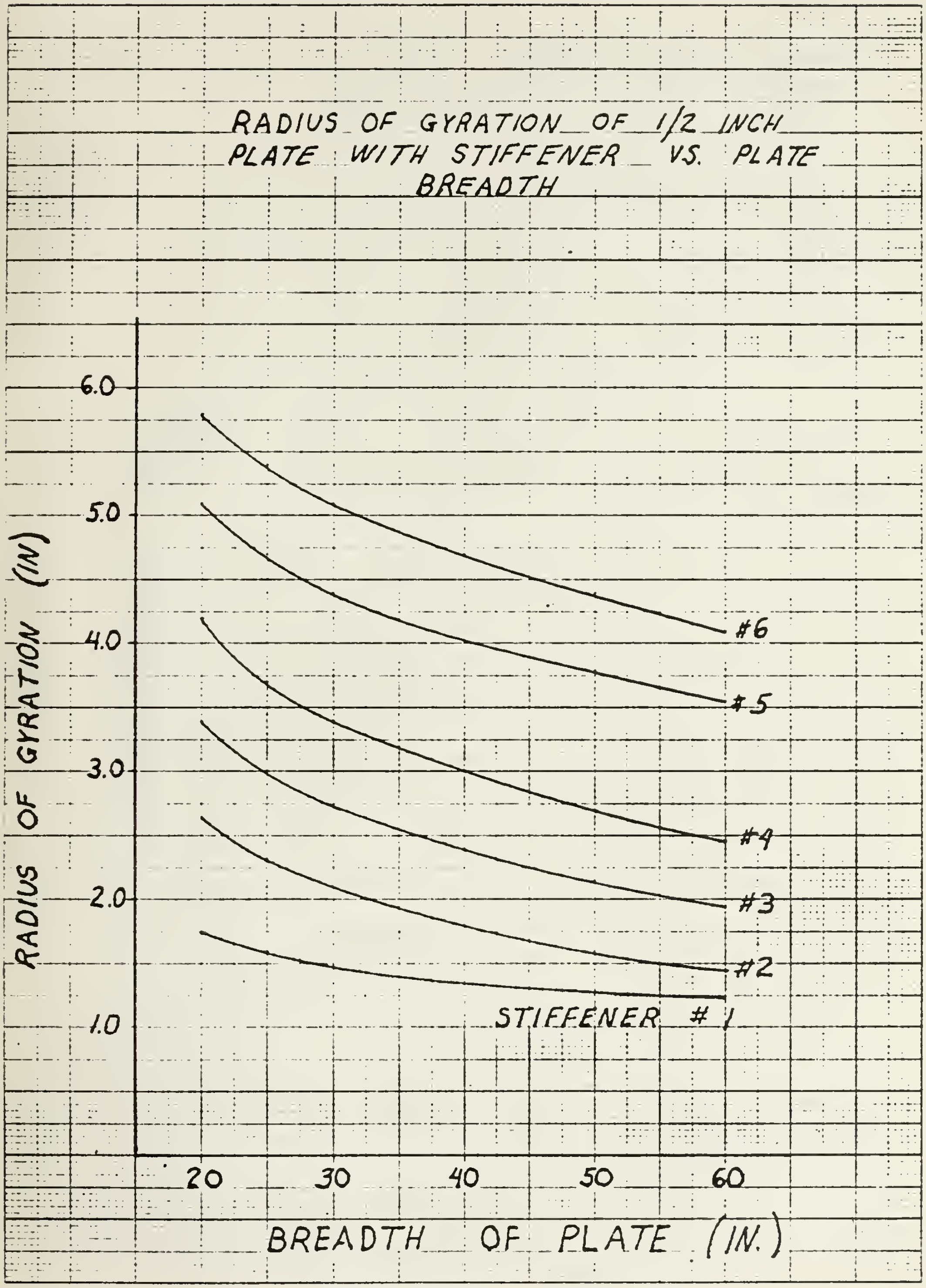

RADIUS OF GYRATION OF 1/2 INCH PLATE WITH STIFFENER VS. PLATE BREADTH
RADIUS OF GYRATION (IN)
6.0
5.0
4.0
3.0
2.0
1.0
#6
#5
#4
#3
#2
STIFFENER #1
20
30
40
50
60
BREADTH OF PLATE (IN.)

FIG. 4

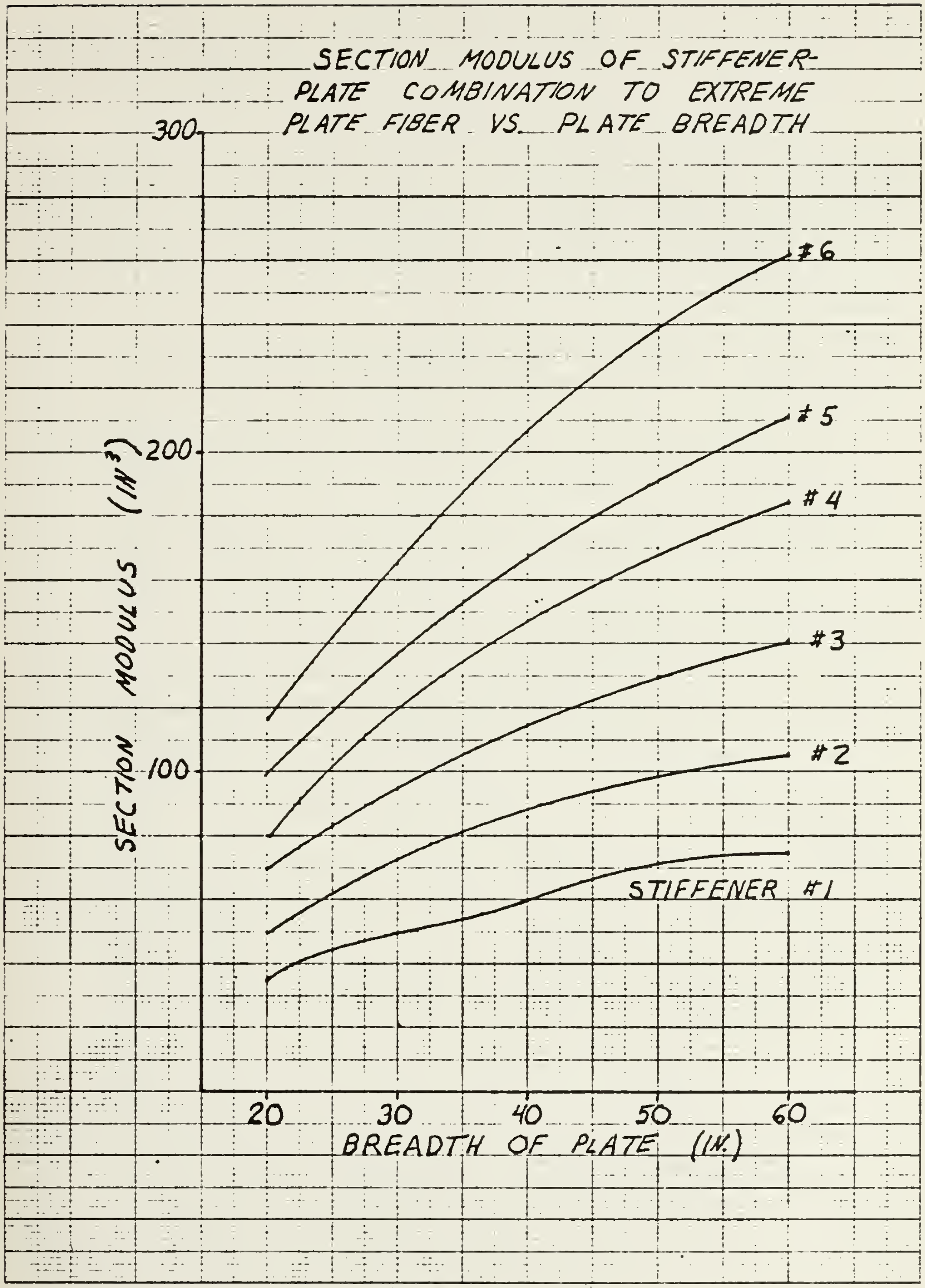
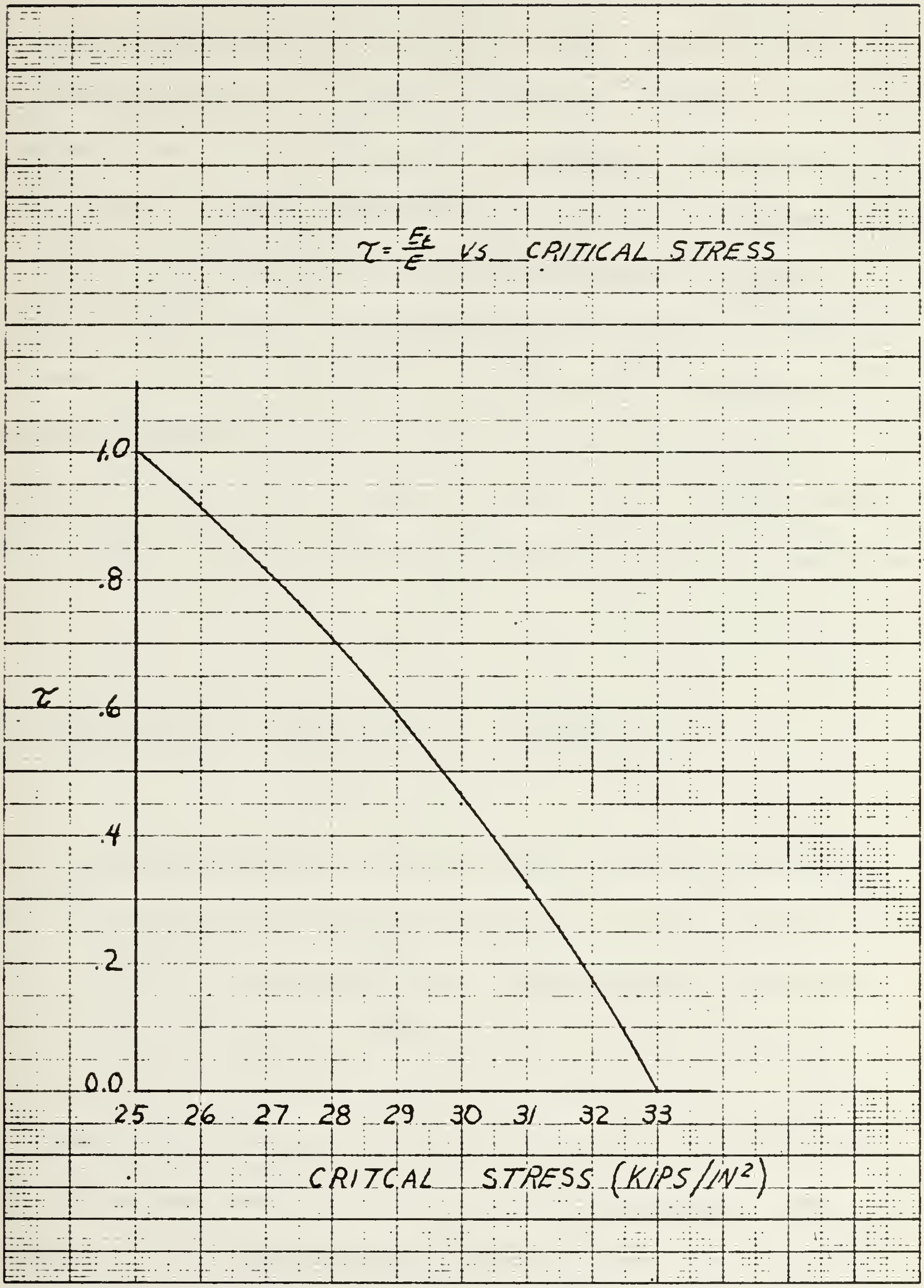

SECTION MODULUS OF STIFFENER-PLATE COMBINATION TO EXTREME PLATE FIBER VS. PLATE BREADTH
300
200
100
SECTION MODULUS (IN³)
#6
#5
#4
#3
#2
STIFFENER #1
20
30
40
50
60
BREADTH OF PLATE (IN.)

FIG. 5



$\tau = \frac{E_t}{E}$ VS. CRITICAL STRESS
1.0
.8
.6
.4
.2
0.0
τ
25
26
27
28
29
30
31
32
33
CRITCAL STRESS (KIPS/IN²)

to the compressive edge (extreme plate fiber) is considered here. The effective breadths used in this paper are those determined by Schade (3). Schade's effective breadth is a function of stiffener configuration (in this case multiple webs), and the ratio of unsupported span length to actual stiffener spacing. (See Appendix D for tabulated values of effective breadths.) In calculating values of Z for different breadths of plating, effective breadths are used rather than actual stiffener spacings. Figure 4 is a plot of Z versus breadth of 1/2 inch plating for various stiffeners.

The next step in developing a useful set of design curves is to substitute equations (2), (3) and (4) into equation (1), and rearrange the variables into some useful form. Direct substitution of equations (2), (3) and (4) into the second form of equation (1) yields:

$$\frac{\pi^2 E \sqrt{\tau_p}}{12(1-\mu^2)} K_p \left(\frac{t}{b}\right)^2 = \frac{\pi^2 E\, \tau_{col}}{\left(\frac{K_c a}{r}\right)^2} \left[1 - \frac{\gamma H b a^2}{K_b\, 144 Z \sigma_y}\right] \tag{5}$$

At first glance this appears to be a very unwieldy equation. Since this study only deals with mild steel, all of the material properties in this equation are constant. The boundary conditions are also known so K_c and K_b are constant. Since critical plate stress and critical column stress are not necessarily equal and the τ values associated with the plate and column are functions of the respective stresses, the two τ values are not equal. (See figure 5 for values of τ.) This leaves the stiffener itself, stiffener spacing, plate thickness, axial load aspect ratio, and the head

of seawater as variables. Since the head of seawater is an early "given variable" in the problem, it may be advantageous to solve equation (5) for head, H. This results in the following:

$$H = \frac{144\sigma_y ZK_b}{\gamma ba^2}\left[1 - \frac{\dfrac{\pi^2 E\sqrt{\tau_p}K_p\left(\frac{t}{b}\right)^2}{12(1-\mu^2)}}{\dfrac{\pi^2 E\tau_{col}}{\left(\frac{K_c a}{r}\right)^2}}\right] \tag{6}$$

The equation was purposely left in this form because it can now be simplified into a more compact form.

$$H = \frac{144\sigma_y ZK_b}{\gamma ba^2}\left[1 - \frac{\sigma_{cr\ p}}{\sigma_{cr\ col}}\right] \tag{7}$$

Equation (7) represents an equality from which a series of useful design curves may be generated. A sample method for determining these curves will now be shown. The following is a list of values for the constant variables for this example:

$E = 3 \times 10^7$ psi

$\mu = .3$

$t = 1/2$ inch

$\gamma = 64$ lbs/ft^3

$a = 12$ feet

$\sigma_y = 33{,}000$ psi

$K_b = 8.0$

$K_c = 1.0$

The next step is to decide upon a set of stiffeners to use.

In naval construction the "T" stiffener is the most common type, and so will be used in this example. To be sure of getting an optimum stiffener arrangement for a given plate thickness and loading arrangement every available stiffener should be tested in equation (7). However, it is not practical to try every stiffener. Webb and Lyons (2) did an investigation in their thesis on all available "T" type stiffeners and came up with six stiffeners that they felt would be adequate for testing in equation (7). The justification for the selection of these stiffeners was that 1) they appeared to give the best structural efficiency, i.e., the highest Z and r values for a given cross sectional area, and 2) they were evenly spaced throughout a broad range. The stiffeners chosen by Webb and Lyons will be identified by number in this paper.

(1)	6 x 4 x 8.5 lb. I-T	(4)	12 x 4 x 14 lb. I-T
(2)	8 x 4 x 10.0 lb. I-T	(5)	12 x 6½ x 27 lb. I-T
(3)	10 x 4 x 11.5 lb. I-T	(6)	14 x 6¾ x 30 lb. I-T

Plots of radius of gyration and section modulus to extreme plate fiber versus plate breadth or stiffener spacing are shown in figures (3) and (4) respectively. It should be noted that effective breadth, not actual stiffener spacing, is used in conjunction with figure (4) when reading off values of section modulus.

Since plate critical stress, $\sigma_{cr\ p}$, is not a function of the size of the stiffener or lateral load, a set of curves resulting from equation (2) determines the stiffener spacing for

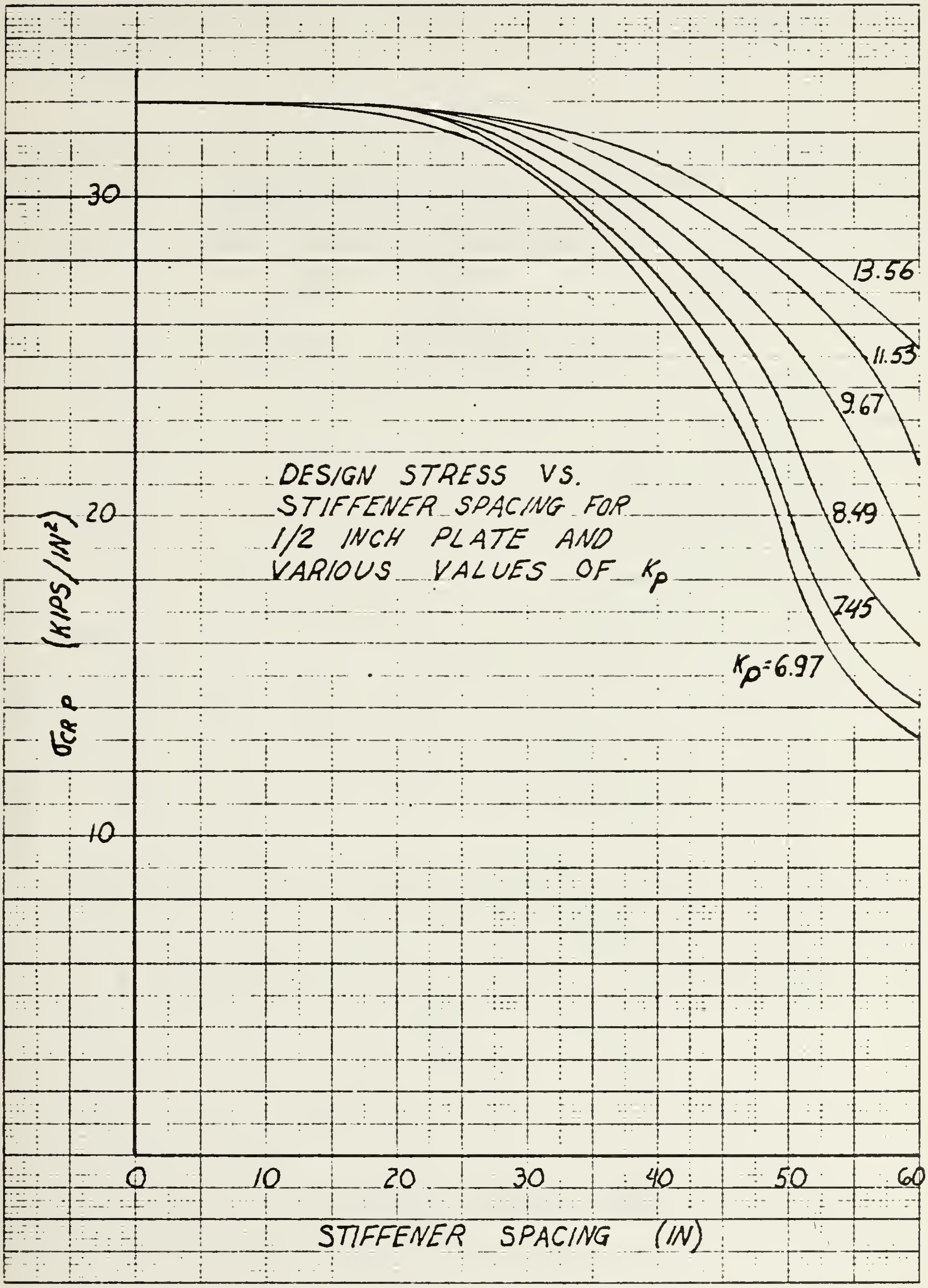
the given loading. For each axial load (or axial stress) and for each axial load aspect ratio there is a maximum stiffener spacing above which local plate buckling will occur. Figure (6) shows these relationships for six values of K_p across a range of pertinent loading aspect ratios.

The second set of curves are used in conjunction with the stiffener spacing determined from the first set of curves to determine the stiffener size. When the stiffener spacing, b, has been determined, this value should be used to enter figures (3) and (4) to find r and Z for each stiffener. These values of r and Z are used in equation (7) to determine the design head of seawater that each combination of stiffener spacing and stiffener can sustain. To develop the second set of curves of design head versus stiffener spacing the procedure described above must be followed for each value of loading aspect ratio and for each stiffener across a complete range of realistic stiffener spacings. Figures (7a-f) show the plots of design head versus stiffener spacing for this example. The points for plotting the curves of figure (7) were determined by a computer program which is listed in Appendix D. Along with the listing of the program is a step by step description of how the inputs are determined and how the computer proceeds to solve equation (7).

Appendices A, B, and C describe the procedures for using figures (6) and (7) to arrive at three optimum stiffener arrangements. Appendix A specifies how to use varying stiffeners and and varying stiffener spacings to arrive at a true optimum

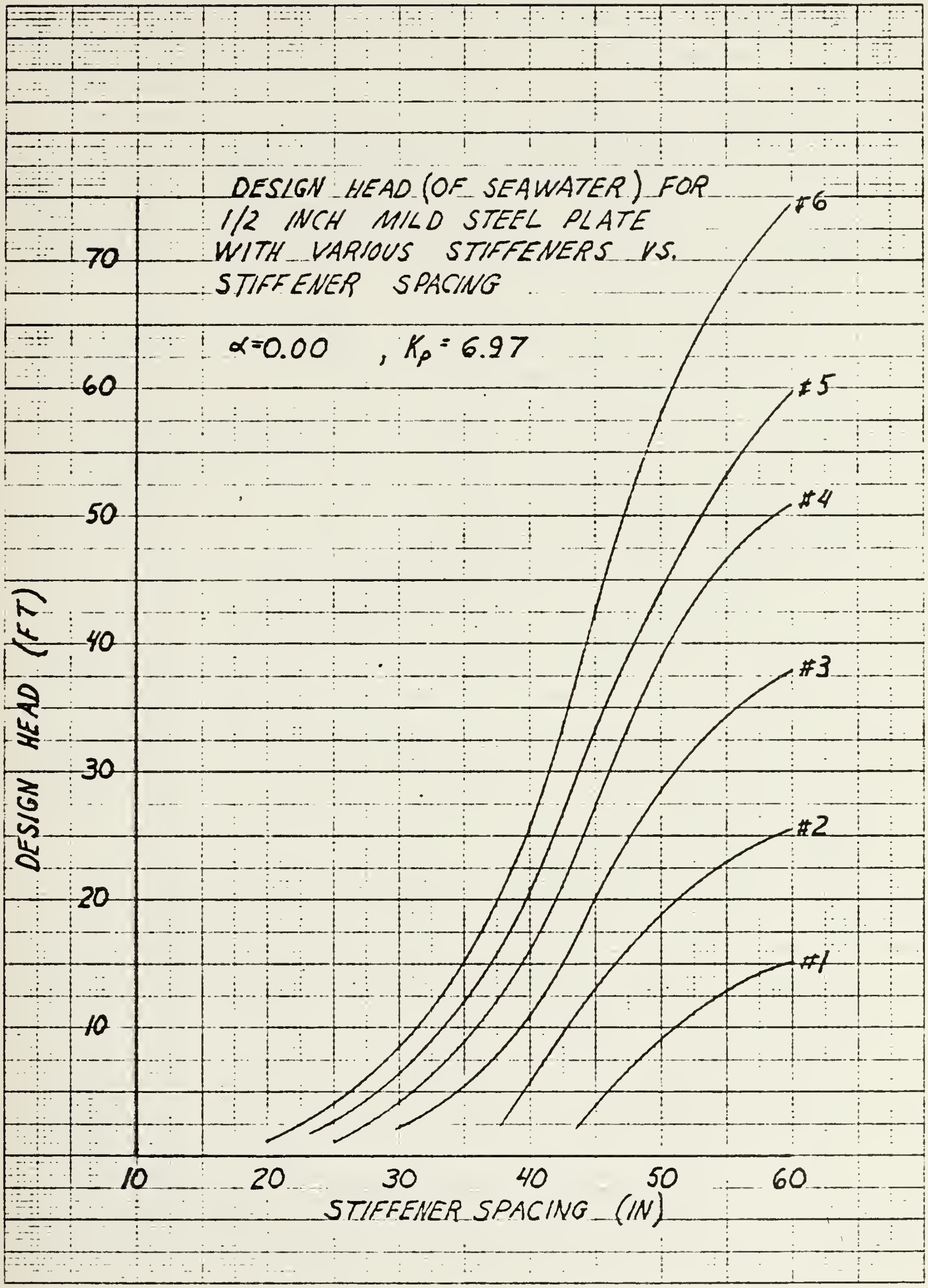
arrangement. Appendix B specifies the use of varying stiffeners at a constant stiffener spacing. Appendix C describes the procedure for using a constant stiffener at varying stiffener spacings. The three examples treat the same gross panel subject to the same load configuration.

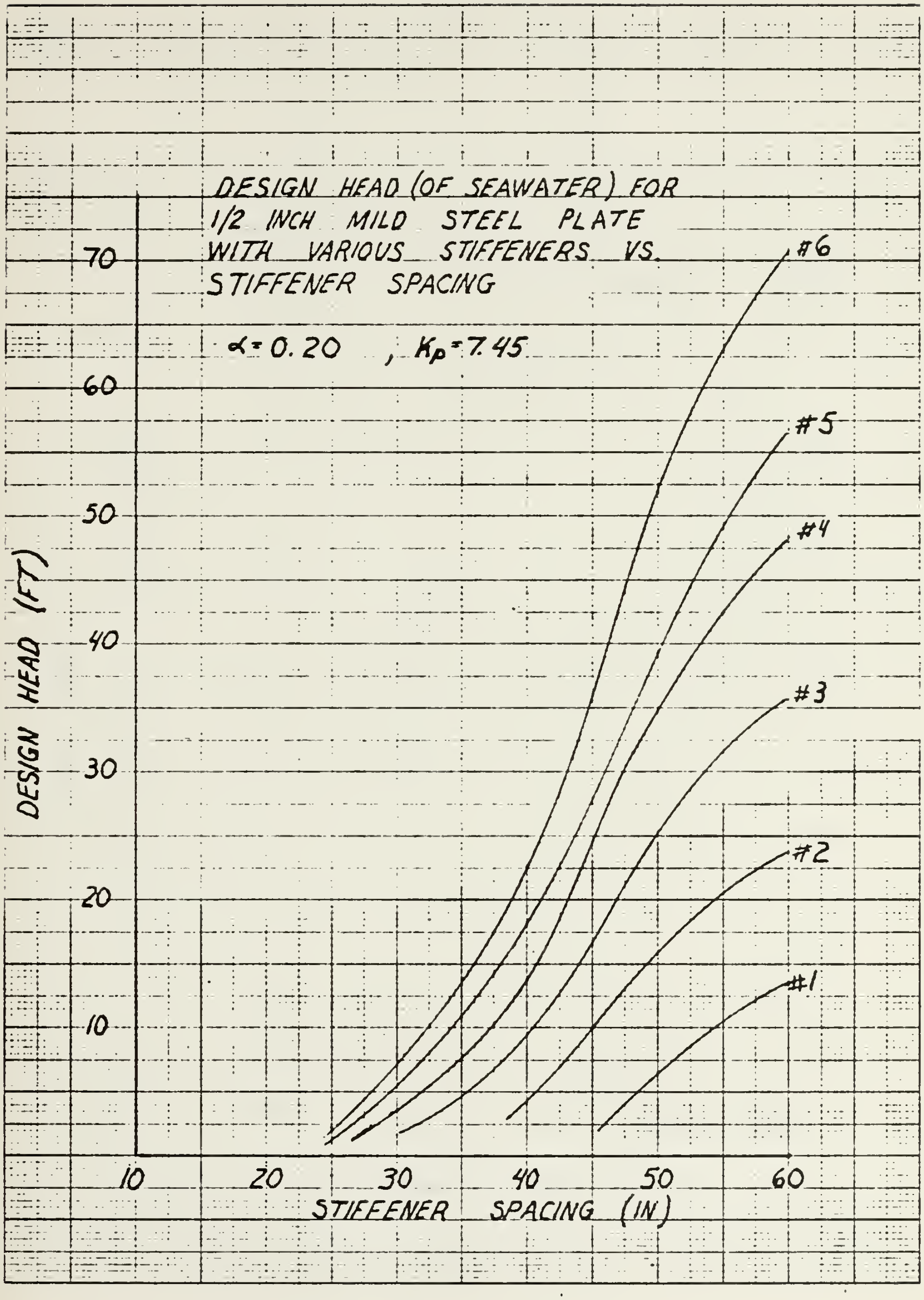
FIG. 6

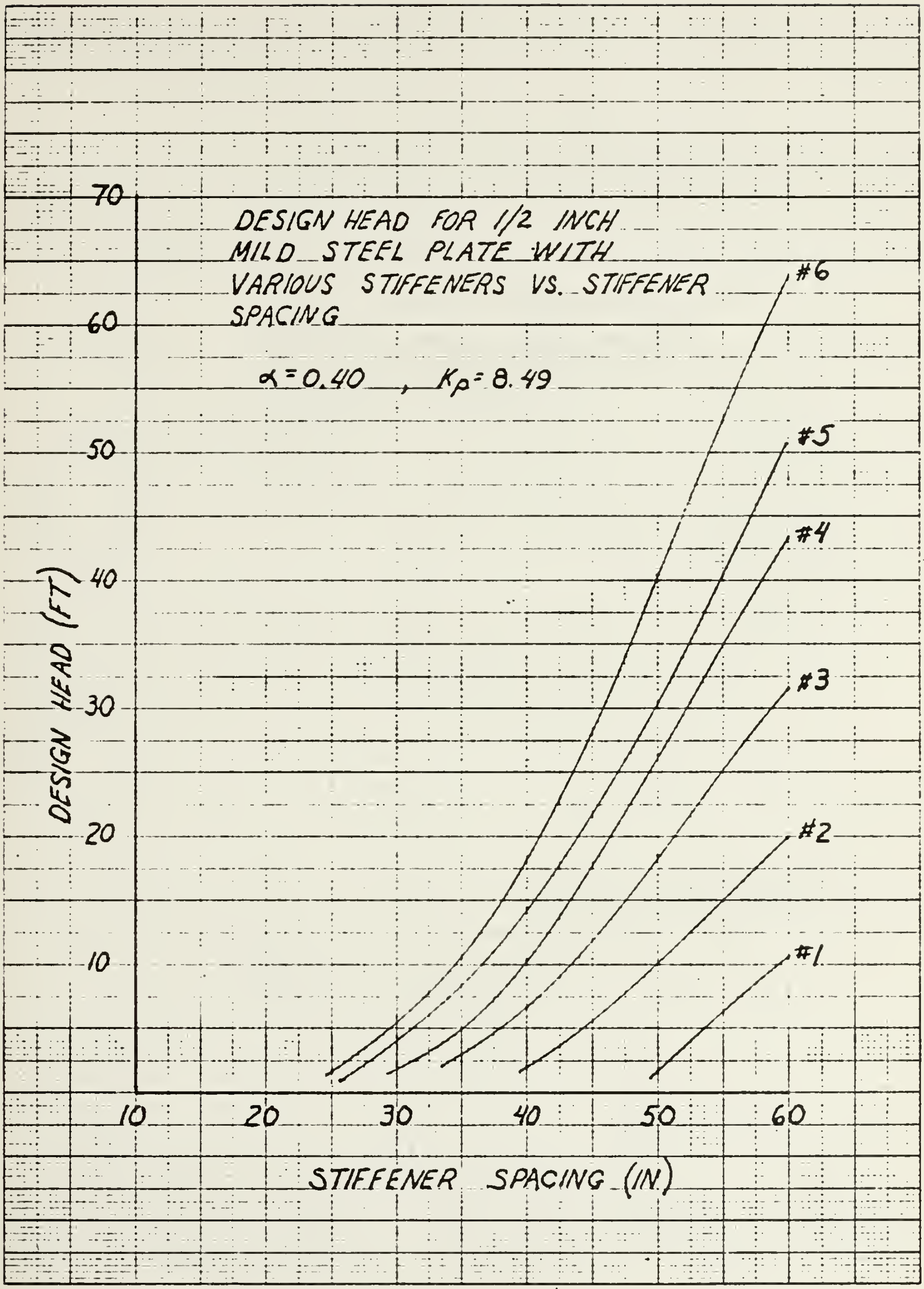


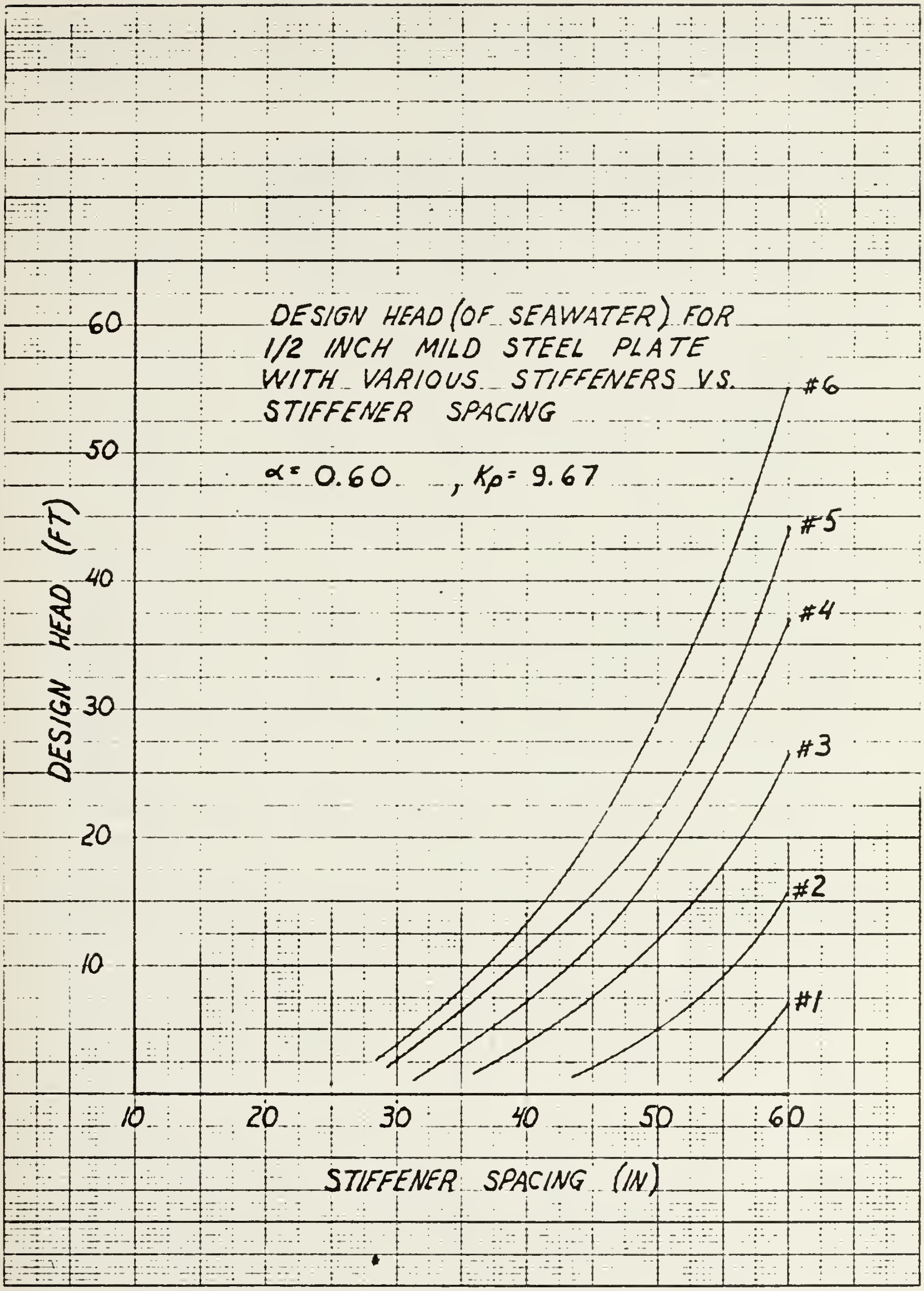
30
20
10
$\sigma_{CR\,P}$ (KIPS/IN²)
DESIGN STRESS VS.
STIFFENER SPACING FOR
1/2 INCH PLATE AND
VARIOUS VALUES OF K_P
13.56
11.53
9.67
8.49
7.45
K_P=6.97
0
10
20
30
40
50
60
STIFFENER SPACING (IN)

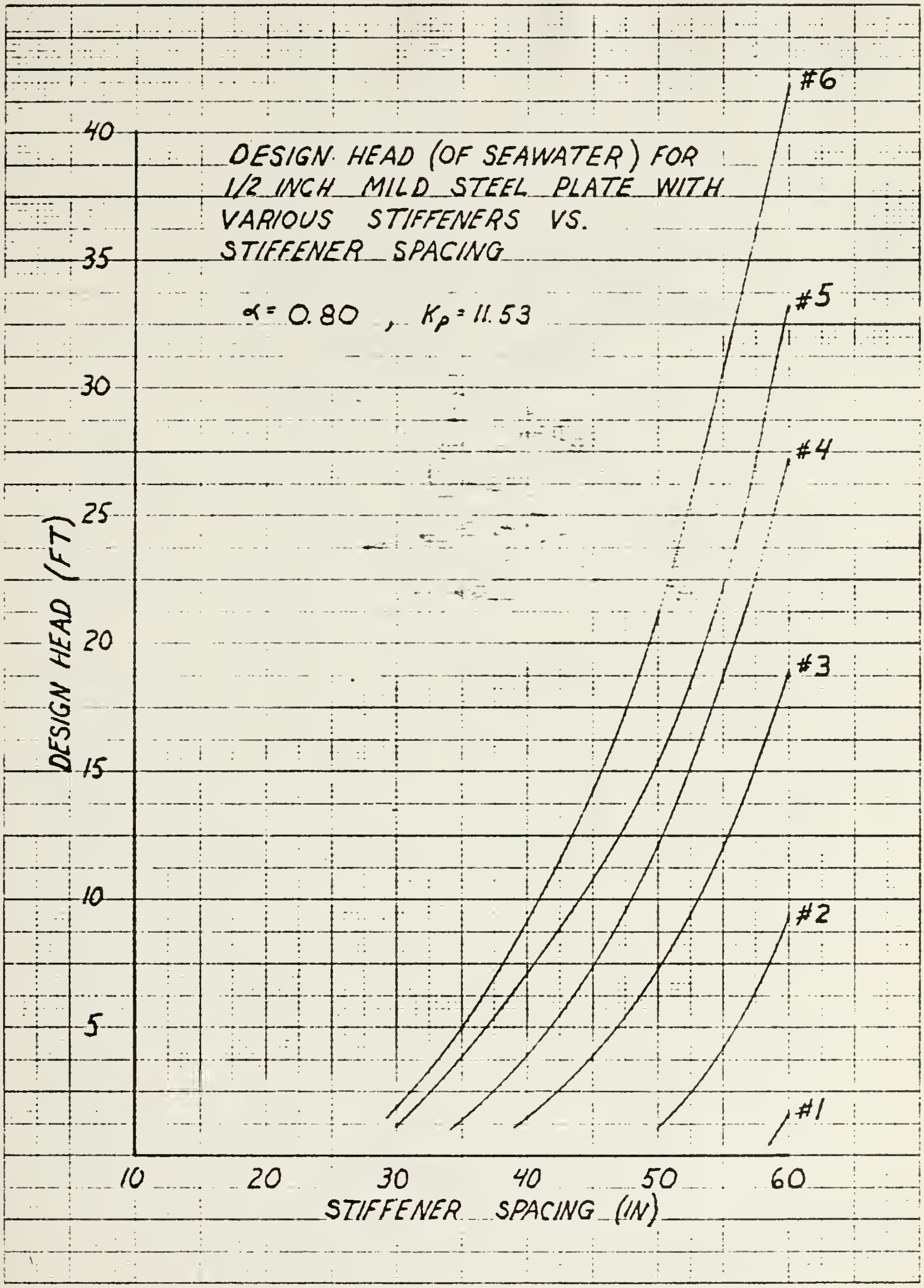
FIG. 7-A

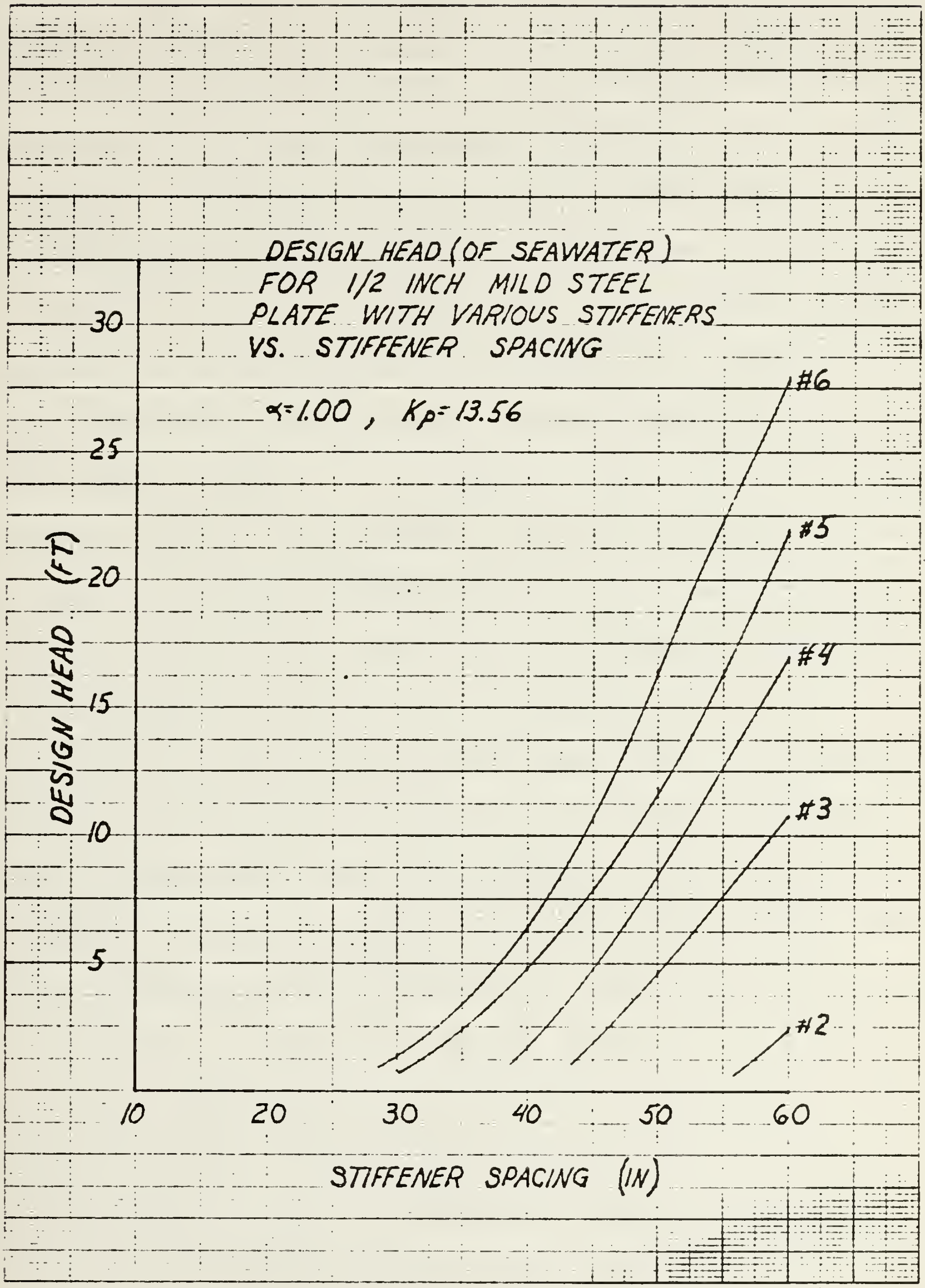

DESIGN HEAD (OF SEAWATER) FOR
1/2 INCH MILD STEEL PLATE
WITH VARIOUS STIFFENERS VS.
STIFFENER SPACING
$\alpha = 0.00$, $K_p = 6.97$
#6
#5
#4
#3
#2
#1
DESIGN HEAD (FT)
70
60
50
40
30
20
10
10
20
30
40
50
60
STIFFENER SPACING (IN)


DESIGN HEAD (OF SEAWATER) FOR
1/2 INCH MILD STEEL PLATE
WITH VARIOUS STIFFENERS VS.
STIFFENER SPACING
α = 0.20 , K_p = 7.45
#6
#5
#4
#3
#2
#1
DESIGN HEAD (FT)
70
60
50
40
30
20
10
10
20
30
40
50
60
STIFFENER SPACING (IN)


DESIGN HEAD FOR 1/2 INCH MILD STEEL PLATE WITH VARIOUS STIFFENERS VS. STIFFENER SPACING
α = 0.40 , Kp = 8.49
#6
#5
#4
#3
#2
#1
70
60
50
40
30
20
10
DESIGN HEAD (FT)
10
20
30
40
50
60
STIFFENER SPACING (IN.)


DESIGN HEAD (OF SEAWATER) FOR
1/2 INCH MILD STEEL PLATE
WITH VARIOUS STIFFENERS VS.
STIFFENER SPACING
α = 0.60 , Kp = 9.67
DESIGN HEAD (FT)
60
50
40
30
20
10
#6
#5
#4
#3
#2
#1
10
20
30
40
50
60
STIFFENER SPACING (IN)


DESIGN HEAD (OF SEAWATER) FOR
1/2 INCH MILD STEEL PLATE WITH
VARIOUS STIFFENERS VS.
STIFFENER SPACING
α = 0.80 , K_p = 11.53
DESIGN HEAD (FT)
40
35
30
25
20
15
10
5
#6
#5
#4
#3
#2
#1
10
20
30
40
50
60
STIFFENER SPACING (IN)


DESIGN HEAD (OF SEAWATER)
FOR 1/2 INCH MILD STEEL
PLATE WITH VARIOUS STIFFENERS
VS. STIFFENER SPACING
α=1.00 , Kp=13.56
#6
#5
#4
#3
#2
DESIGN HEAD (FT)
30
25
20
15
10
5
10
20
30
40
50
60
STIFFENER SPACING (IN)

III. Results

The design curves of figures (6) and (7) are the results of the procedures outlined in the previous section. These curves aid the designer in his selection of an optimum stiffener-plate combination subject to the constraints mentioned. It is noted that the optimum resulting from one set of curves represents an optimum arrangement for only one plate thickness. A set of curves must be generated for all standard plate sizes and the optimum configuration for each size must be determined. Only the stiffener-plate combination of the least weight using all sizes of plate is a true optimum. It is commonly found that arrangements using thinner plates with narrower stiffener spacings tend to be lighter than those using thicker plates with wider spacings. In practice, a minimum plate thickness is frequently specified which may preclude a true optimum arrangement.

The curves of figure (6) follow trends that are expected. For stresses above 25,000 psi the tangent modulus criteria is used because the elastic limit is exceeded although buckling has not begun. The term τ_p determines the shape of the curves above 25,000 psi as it is a function of stress above this value. Below 25,000 psi, τ_p equals 1.0 so the shape of the curves is hyperbolic as dictated by Bryan's equation. The more extreme the loading condition (lower values of K_p) the lower the critical stress for panels of equal dimensions. Another phenomenon illustrated by these curves is that as the distance between stiff-

eners is increased the critical plate stress is decreased. Figure (6) is used to determine the local plate breadth based on the axial stress alone.

At first glance the curves in figure (7) may seem paradoxical. It appears that the larger stiffener spacings can sustain a larger head of seawater. However, it must be remembered that larger stiffener spacings imply lower values of axial stress, and that each point on these curves represents an optimum plate-stiffener arrangement.

When using figure (7) it is noted that the intersection of stiffener spacing and maximum design head that the local panel supports does not always lie directly on one of the curves. Since the stiffeners plotted have the best structural efficiencies, any stiffener whose structural properties lie between the two in question will probably not yield a much lighter arrangement. In this case the use of the larger stiffener is the logical choice.

The stiffener arrangements determined in Appendices A, B, and C offer some interesting results. As hypothesized, in Appendix A where the stiffener size and spacing vary with the load, the stiffener size decreases and the local panel size increases as the lateral and axial loads decrease. The larger axial loads dictate the smaller panel sizes and the larger heads of seawater dictate larger stiffener sizes. An interesting variation of the loading on the plate would be to invert either the axial or lateral load, thus having the maximum axial load acting at the same edge as the minimum lateral load. This configuration

is not unrealistic as it represents the loading on a panel of shell plate below the waterline of a ship but above the neutral axis when the ship is sagging. The optimum stiffener arrangement for a panel subject to this type of loading would have larger stiffeners acting with the larger heads of seawater and smaller stiffener spacings acting with the larger axial loads.

The example worked out in Appendix B treats the same panel and load as in Appendix A, but the design is limited to a constant stiffener spacing. In this arrangement the stiffener spacing for the whole panel is determined by the maximum axial load. This spacing will be smaller than an optimum spacing for the smaller axial loads. In other words, with respect to axial load the stiffener-plate combination is over-designed. Since the curves of figure (7) are plotted using maximum values of stiffener spacing corresponding to each axial load, it is this value, not the constant spacing already determined, that is used in conjunction with the head of seawater to determine the correct stiffener for each local panel. If the actual stiffener spacing determined by the maximum axial load on the gross panel was used to enter figure (7) the resulting stiffener would be too large. It must be remembered that the smaller stiffener spacings automatically imply larger axial stresses so to use figure (7) with the actual predetermined spacing, and not the spacing based on local axial load, would result in an overly conservative design. The penalty paid by constraining the arrangement to a constant stiffener spacing is that more stiffeners must be

used than that of the optimum design.

Appendix C again pertains to the same panel and loading configuration as was described in Appendix A. The design for this example is constrained to using only one stiffener. The stiffener size for this arrangement is determined by the head of seawater acting in conjunction with the stiffener spacing. Since the optimum stiffener spacing is determined by the axial load alone, it is the maximum applied lateral load which determines which stiffener to use for this example. The optimum panel sizes will be identical to those determined in the example of Appendix A, and the largest stiffener determined from the maximum head of seawater is used for each local panel. It is evident that the panels subjected to the smaller lateral loads will be stiffened by a stiffener larger than the one in the case of the optimum design.

A final remark on the results of these three examples is in order. It must be noted that when entering figure (6) the maximum local axial stress is used and when entering figure (7) the maximum local head of seawater is used. The use of the maximum local loads instead of average load, or load at the centroid (of the linearly varying load) tends to make each of the arrangements outlined somewhat conservative.

IV. Conclusions

1. It cannot be determined analytically whether or not a plate-stiffener arrangement designed according to this thesis is totally valid. However, all assumptions are based on established engineering principles, so the procedures developed should yield a sound design.

2. The plate-stiffener arrangements designed according to the procedures outlined in this paper are conservative. <u>Maximum</u> local axial stress and <u>maximum</u> local head of seawater are used to determine stiffener spacing and size respectively.

3. For a panel subjected to linearly varying axial and lateral loads only an arrangement utilizing various stiffeners and spacings is optimum from a minimum weight standpoint. An arrangement constrained to use only one stiffener size is overdesigned with respect to the number of stiffeners.

4. The design curves developed in this thesis may be valuable to ship designers for a least-weight stiffener arrangement for a ship's side shell plating.

V. Recommendations

1. That a complete set of design curves be generated, using a computer program similar to the one used in this paper, which cover all standard plate thicknessess and a variety of span lengths.

2. That any further design curves generated must cover stiffener spacings well beyond 60 inches as was done in this thesis.

3. That a complete side shell of a midship section be designed utilizing these design curves and that this midship section be compared to a midship section designed in a conventional manner.

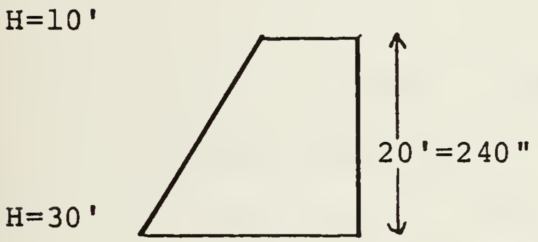
4. That a study be carried out to investigate the feasibility of designing and fabricating sections of side shell plating utilizing various stiffeners and spacings.

5. That the effects of shear loading be investigated and possibly accounted for in the interaction formula.

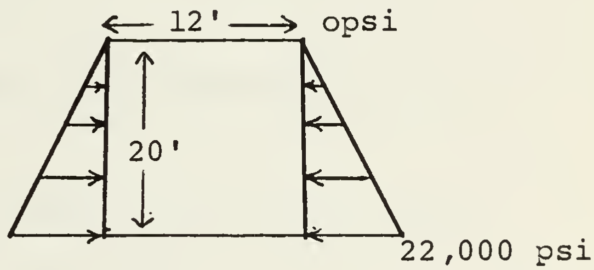
6. That the feasibility of running the stiffeners perpendicular to the axial load be investigated.

An example of the use of the design curves, figures (6) and (7), for an optimum weight stiffener arrangement specifying various stiffeners and various stiffener spacings will be shown. It is again noted that this example pertains only to the case for which the design curves in this thesis apply. The entire process has to be followed for all allowable sizes of shell plating to come up with an actual optimum weight stiffener arrangement, as this example solution yields an optimum arrangement for only one size of plate.

Consider a 1/2 inch mild steel panel loaded as below:



lateral load of seawater



axial load due to ship bending as a beam

The following step by step procedure describes the method of using figures (6) and (7) to optimally stiffen this gross panel of side shell plating.

1) Enter figure (6) with $\sigma_{cr\ p}$ of 22,000 psi and α of 0.0. This yields a stiffener spacing of 47 inches.

2) The new α using stiffener spacing of 47 inches is 47/240= .195.

3) Enter figure (6) again with $\sigma_{cr\ p}$ of 22,000 psi and α of .195.

4) The new α using stiffener spacing of 48 inches is 48/240 = .2.

5) This iterative procedure has rapidly converged for a stiffener spacing for the first local panel of 48 inches. (It is noted that this is actually a conservative spacing because the σ_{crp} used to enter figure (6) is larger than the average axial stress over the plate panel.)

6) With α of .2 use figure (7-b). Enter with stiffener spacing of 48 inches and design head (maximum head corresponding to bottom local panel) of 30 feet and check intersection.

7) On figure (7-b) the intersection lies between the curves of stiffener #3 and stiffener #4. For a conservative engineering design choose the larger stiffener or stiffener #4 for the first local panel.

8) The procedure now repeats itself for choosing the second panel size and panel stiffener.

9) The axial stress or $\sigma_{cr\ p}$ 48 inches up from the bottom of the panel is $22{,}000 \times \frac{240-48}{240} = 17{,}600$ psi and the head of seawater 48 inches up from the bottom of the panel is

$$(30-10) \times \frac{240-48}{240} + 10 = 26 \text{ feet.}$$

10) Enter figure (6) with $\sigma_{cr\ p}$ of 17,600 psi and α of 0.0. This yields a stiffener spacing of 51 inches.

11) The new α using stiffener spacing of 51 inches is 51/(240-48) = .266.

12) Entering figure (6) again with $\sigma_{cr\ p}$ of 17,600 and α of .266 yields a new stiffener spacing of 53 inches.

13) Iterating once more shows that the stiffener spacing is 53 inches. $\alpha = 53/192 = .276$.

14) Since there is no plot in figure (7) corresponding to α of .276, it is conservative to enter figure (7-b) for α of .2 with a stiffener spacing of 53 inches and design head of 26 feet. The intersection lies just below the curve for stiffener #3 so choose stiffener #3 for the second local panel.

15) The procedure continues to repeat itself by essentially decreasing the size of the gross panel until the loads are small enough that the remaining panel requires no stiffening.

16) The preceding procedure must be carried out for all standard plate sizes, and the weights of each resulting stiffener-plate combination must be compared. The combination with the least weight represents the true optimum using the least weight criterion of this thesis.

Appendix B

Consider the gross panel loaded in Appendix A, in the same manner. From a fabrication standpoint it may be more feasible to stiffen the panel with a constant stiffener spacing but using different stiffeners. Obviously this will not be as effective from a weight standpoint as the arrangement described in Appendix A with varying stiffener spacings and stiffeners, but because it may be more practical it is of interest to describe the procedure for optimally stiffening the gross panel with a constant stiffener spacing.

1) Assume α to be 0.0 and enter figure (6) with $\sigma_{cr\ p}$ of 22,000 psi. This yields a stiffener spacing of 47 inches.

2) Continuing to repeat the procedure outlined in Appendix A yields a stiffener spacing of 48 inches. This will be the stiffener spacing for each local panel.

3) For α of $48/240 = .2$ enter figure (7-b) with design head of 30 feet and stiffener spacing of 48 inches.

4) Following the same logic as in Appendix A choose stiffener #4.

5) With a stiffener spacing of 48 inches the second local panel will have $\alpha = 48/(240-48) = .25$.

6) The maximum design head acting on the second panel is $\left[(30-10) \times \frac{240-48}{240}\right] + 10 = 26$ feet.

7) The maximum axial stress acting on the second panel

is 22,000 x $\frac{240-48}{240}$ = 17,600 psi. From figure (6) this yields an optimum (not actual) stiffener spacing of 53 inches.

8) Using figure (7-b) with design head of 26 feet and stiffener spacing of 53 inches, choose for the second local panel stiffener #3.

9) The arrangement specified thus far resembles very much that determined up to this point in Appendix A. However, because of the reduced stiffener spacing the differences in the arrangements will increase after this point.

10) The above procedure repeats itself three more times until each local panel of 48 inch width has been considered. It is important to remember to use the optimum stiffener spacing corresponding to the local values of α and $\sigma_{cr\ p}$ from figure (6), and not the actual value of stiffener spacing in figure (7), in conjunction with local head of seawater to determine the correct stiffener.

Appendix C

One last stiffener arrangement which may be of interest for the same reasons as described in Appendix B is the use of one stiffener at various stiffener spacings. The gross panel and the loading configuration is once again the same as the panel considered in Appendices A and B. The procedure for determining an optimum stiffener arrangement with a constant stiffener size is described below.

1) Assume α of 0.0 and enter figure (6) with a $\sigma_{cr\ p}$ of 22,000 psi. Read off a stiffener spacing of 47 inches.

2) Continuing to iterate identically as in Appendix A yields a stiffener spacing of 48 inches. This will be the size of the first local panel.

3) For α of 48/240 = .2 enter figure (7-b) with a design head of 30 feet and stiffener spacing of 48 inches.

4) Following the same logic as in Appendix A choose stiffener #4. This stiffener will be used in conjunction with all subsequent local panels.

5) The axial stress 48 inches up from the bottom of the panel is 22,000 x $\frac{240-48}{240}$ = 17,600 psi and the head of seawater at the same location is $\left[(30-10) \times \frac{240-48}{240}\right] + 10 = 26$ feet.

6) Enter figure (6) with $\sigma_{cr\ p}$ of 17,600 psi and an assumed α of 0.0 yields a stiffener spacing of 51 inches.

7) Following the same procedure as in Appendix A yields

a second panel breadth of 53 inches. (Because stiffener spacing or panel breadth is dictated by the axial load, the local panels for this arrangement will be identical to those of the arrangement described in Appendix A.)

8) Use stiffener #4 to stiffen all local panels.

Appendix D

A computer program was written to solve the equation for design head, equation (7), using the large number of variables required. Some preliminary hand calculations must be performed to determine input values for the computer program. This program has been written for the specific case of six stiffeners, six spacings, and six axial load aspect ratios. However, the program can easily be modified to handle more than six of any of these variables. The inputs for the program are the six stiffener spacings, the six axial load aspect ratios and the corresponding K_p values for plate critical stress, values of section modulus for each stiffener acting with each stiffener spacing, values of critical column stress for each stiffener acting with each stiffener spacing, and values of plate critical stress for each loading aspect ratio of each stiffener spacing. The following is a step by step procedure for determining the inputs to this computer program:

1) Choose six useful stiffener spacings.

2) From Schade's paper read off values of ratio of effective breadth to actual spacing for each value of stiffener spacing and given unsupported span length.

3) Determine values of effective breadth.

4) From figure (4) read off values of section modulus for each stiffener at each effective breadth.

5) From figure (3), read off values of radius of gyration

for each stiffener at each actual stiffener spacing.

6) Using equation (3) and values of radius of gyration determined in step 5, calculate critical column stress for each stiffener at each stiffener spacing.

7) From figure (6) read off values of critical plate stress for each axial load aspect ratio at each stiffener spacing.

The arrangement of these inputs in the data deck is as follows: (All values entered six at a time are in ascending order of stiffener spacing)

Card 1 - Six values of stiffener spacing (6 F7.2)

Card 2 - First loading aspect ratio and corresponding K_p (2 F4.2)

Card 3 - Six values of plate critical stress corresponding to first axial load aspect ratio (6 F7.2)

Card 4 - Six values of section modulus for first stiffener (6 F7.2)

Card 5 - Six values of critical column stress for first stiffener (6 F7.2)

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Card 15 - Six values of critical column stress for sixth stiffener (6 F7.2)

Card 16 - Second loading aspect ratio and corresponding K_p (2 F4.2)

Card 17 - Six values of plate critical stress corresponding to second axial load aspect ratio (6 F7.2)

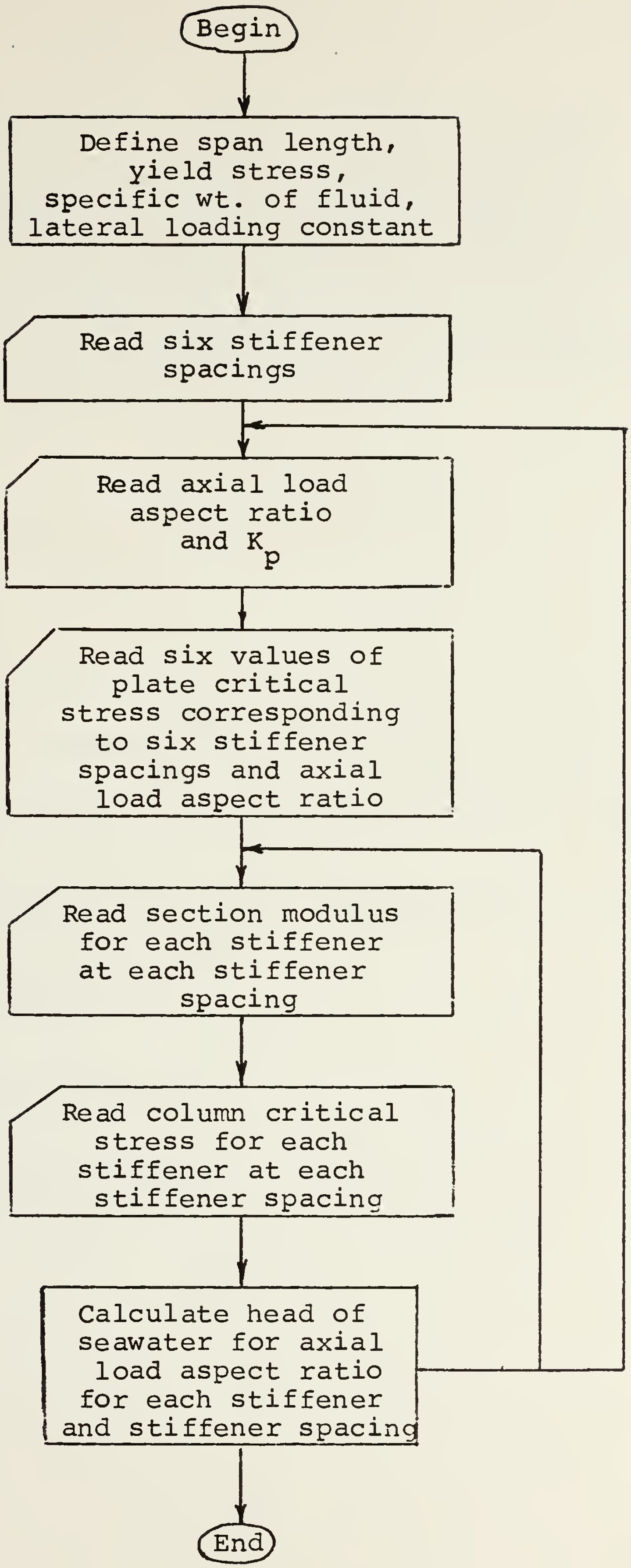
.
.
.

Card 25 - Six values of plate critical stress corresponding

to sixth axial load aspect ratio (6 F7.2)

The inputs to the computer program are listed in tabular form. Also following are a macroscopic flow chart, a listing of the program, and a sample output.

Stiffener spacings (in.)		20	25	30	40	50	60
Effective breadths (in.)		19.4	23.8	27.9	35.6	42	46.8
Section modulus (in^3) for each stiffener from figure (4) based on effective breadths	#1	35	44	50	60	72	75
	#2	50	63	73	86	99	106
	#3	70	84	95	115	130	141
	#4	90	104	120	147	170	185
	#5	100	120	137	168	184	212
	#6	117	142	165	205	239	263
Radius of gyration (in.) for each stiffener from figure (3) based on actual stiffener spacings	#1	82.3	90	97.5	106	112	116
	#2	54.4	62.3	68.5	80	90	99.5
	#3	42.4	48	52.4	60	67	74
	#4	34.3	38.9	42.4	48	53.4	58.3
	#5	28.2	30.7	32.8	35.9	38.1	40.6
	#6	24.9	26.7	28.2	30.6	32.8	35.2
Plate critical stress for each axial load aspect ratio from figure (6) and equation (2)	0.0	32,500	31,900	30,800	26,800	18,700	13,100
	.2	32,800	32,200	31,000	27,500	20,200	14,100
	.4	32,800	32,300	31,400	28,500	22,900	15,900
	.6	32,800	32,500	31,800	29,400	25,500	18,100
	.8	32,800	32,600	32,200	30,400	27,500	21,600
	1.0	32,800	32,600	32,300	31,100	28,600	25,200
Critical column stress for each stiffener from equation (3)	#1	29,200	28,000	27,000	25,000	24,000	22,700
	#2	31,500	30,800	30,400	29,500	28,000	26,500
	#3	32,000	31,700	31,600	31,000	30,500	30,000
	#4	32,500	32,200	32,000	31,600	31,700	31,200
	#5	32,600	32,500	32,500	32,400	32,200	32,150
	#6	32,700	32,650	32,600	32,500	32,500	32,400


Begin
Define span length, yield stress, specific wt. of fluid, lateral loading constant
Read six stiffener spacings
Read axial load aspect ratio and K_p
Read six values of plate critical stress corresponding to six stiffener spacings and axial load aspect ratio
Read section modulus for each stiffener at each stiffener spacing
Read column critical stress for each stiffener at each stiffener spacing
Calculate head of seawater for axial load aspect ratio for each stiffener and stiffener spacing
End

```
C     THIS PROGRAM CALCULATES THE DESIGN HEAD OF SEA WATER THAT A
C     STIFFENED PLATE SUBJECTED TO A LINEARLY VARYING AXIAL LOAD CAN
C     SUSTAIN.  THE STIFFENED PLATE IS TREATED AS A BEAM-COLUMN BY
C     MAKING USE OF A COMMON INTERACTION FORMULA.  THE CALCULATIONS
C     ARE FOR A PLATE OF CONSTANT THICKNESS OF ONE HALF INCH, CONSTANT
C     UNSUPPORTED SPAN OF TWELVE FEET, AND MADE OF MILD STEEL.  THE
C     VARIABLES ARE STIFFENER SIZE AND SPACING AND THE AXIAL LOADING
C     CONDITION.  THE PLATE IS CONSIDERED CLAMPED IN THE DIRECTION OF THE
C     STIFFENERS.
      DIMENSION B(6),SCP(6),H(6),Z(6,6),SCC(6,6)
      SY=33000.
      CL=8.
      G=64.
      A=12.
      C1=(SY*CL)/(G*A*A)
      M=0
      READ(8,100)(B(I),I=1,6)
100   FORMAT(6F7.2)
10    M=M+1
      IF(M.EQ.7) STOP
      READ(8,200) AL,CCP
200   FORMAT(2F4.2)
      WRITE(5,300)
300   FORMAT('1',10X,'DESIGN HEAD(FT) FOR VARIOUS STIFFENERS AND SPACING
     C (IN)')
      WRITE(5,400) AL,CCP
400   FORMAT('0',10X,'ALPHA=',F4.2,10X,'K(PLATE CRITICAL STRESS)='
     C,F5.2)
      WRITE(5,500)(B(I),I=1,6)
500   FORMAT('0',23X,'B=',F6.2,',B=',F6.2,',B=',F6.2,',B=',F6.2,
     C',B=',F6.2,',B=',F6.2)
      WRITE(5,600)
600   FORMAT(10X,'STIFFENER NUMBER')
      READ(8,100)(SCP(I),I=1,6)
      N=0
20    N=N+1
      IF(M.GE.2) GO TO 50
      READ(8,100)(Z(N,I),I=1,6)
      READ(8,100)(SCC(N,I),I=1,6)
50    DO 30 J=1,6
30    H(J)=C1/B(J)*(1.-SCP(J)/SCC(N,J))*Z(N,J)
      WRITE(5,700)N,(H(J),J=1,6)
700   FORMAT(10X,I2,10X,F7.2,2X,F7.2,2X,F7.2,2X,F7.2,2X,F7.2,2X,F7.2)
      IF(N.NE.6) GO TO 20
      GO TO 10
      END
```

DESIGN HEAD(FT) FOR VARIOUS STIFFENERS AND SPACING (IN)

ALPHA=0.00 K(PLATE CRITICAL STRESS)= 6.97

STIFFENER NUMBER	B= 20.00	B= 25.00	B= 30.00	B= 40.00	B= 50.00	B= 60.00
1	-5.67	-7.02	-6.72	-3.09	9.11	15.14
2	-2.27	-2.58	-0.92	5.64	18.84	25.59
3	-1.57	-0.61	2.30	11.16	28.81	37.92
4	0.00	1.11	4.30	15.99	39.39	51.24
5	0.44	2.54	6.84	20.79	44.20	59.97
6	1.03	3.74	8.70	25.75	58.14	74.30

DESIGN HEAD(FT) FOR VARIOUS STIFFENERS AND SPACING (IN)

ALPHA=1.00 K(PLATE CRITICAL STRESS)=13.50

STIFFENER NUMBER	B= 20.00	B= 25.00	B= 30.00	B= 40.00	B= 50.00	B= 60.00
1	-6.18	-8.28	-9.37	-10.48	-7.91	-3.94
2	-2.96	-4.22	-4.36	-3.34	-1.22	2.45
3	-2.51	-2.73	-2.01	-0.27	4.64	10.77
4	-1.19	-1.48	-1.07	1.67	8.59	16.99
5	-0.88	-0.42	0.81	4.83	11.79	21.88
6	-0.51	0.25	1.45	6.32	16.43	27.93

Summary of program variables:

SY = yield stress
CL = lateral load coefficient
G = specific weight of seawater
A = unsupported span length
M = counter for number of axial load aspect ratios
B = stiffener spacing
I = counter for number of stiffener spacings
AL = axial load aspect ratio
CCP = plate critical stress coefficient
N = counter for number of stiffeners
SCP = plate critical stress
Z = section modulus of plate-stiffener combination
SCC = critical column stress

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